

# **TEHNICI DE CLUSTERING IN MODELAREA FUZZY**

**Aplicații în sistemele electroenergetice**

# Obiective

Folosirea modelelor fuzzy îmbunătățite prin tehnici de clustering în calcule cu mărimi incerte.

## Aplicații în sistemele electroenergetice

- ✓ *Estimarea pierderilor de putere/energie*
- ✓ *Determinarea profilurilor tip de sarcină*
- ✓ *Determinarea profilelor optime de reglaj a tensiunii.*

## Conducerea optimală a sistemelor electroenergetice presupune:

- Formularea problemei;
- Construirea modelului matematic:

$$\text{FO } \min F(X, U)$$

Resticții

$$h_i(X_1, X_2, \dots; U_1, U_2, \dots, U_m) = 0, \quad i = \overline{1, m_h}$$

$$g_i(X_1, X_2, \dots, U_1, U_2, \dots, U_m) \leq 0, \quad i = \overline{1, m_g}$$

$$X_j \geq 0, \quad j = 1, \dots, n$$

- Determinarea soluției optime.

### Difficulties:

- Multiobjective modeling;
- Constraints handling;
- Uncertainties modeling (loads, costs).

## □ 1. Ce este clusterizarea?

Tehnicile de clustering (grupare spațială) reprezintă tehnici speciale de aranjare a datelor de intrare pe baza dispunerii spațiale a vectorilor corespunzători.

Un obiect/element = un vector de măsurători / caracteristici:

$$x = (x_1, \dots, x_d). \quad x_i = \text{caracteristici / attribute}$$

Proximitatea elementelor este măsurată de obicei printr-o funcție de distanță definită de perechi de elemente. Măsurile de asemănare pot fi folosite și pentru a caracteriza similitudinea conceptuală dintre două sau mai multe elemente.

1. Clusterizarea ierarhică: - aglomerativă;

- divizivă;

2. Clusterizarea K-medii.

Exemple de modele:

p – Atlanta

q – Huston

r – Denver

s – New York

t – Washington

Atribute:

Populație

Suprafață

PIB

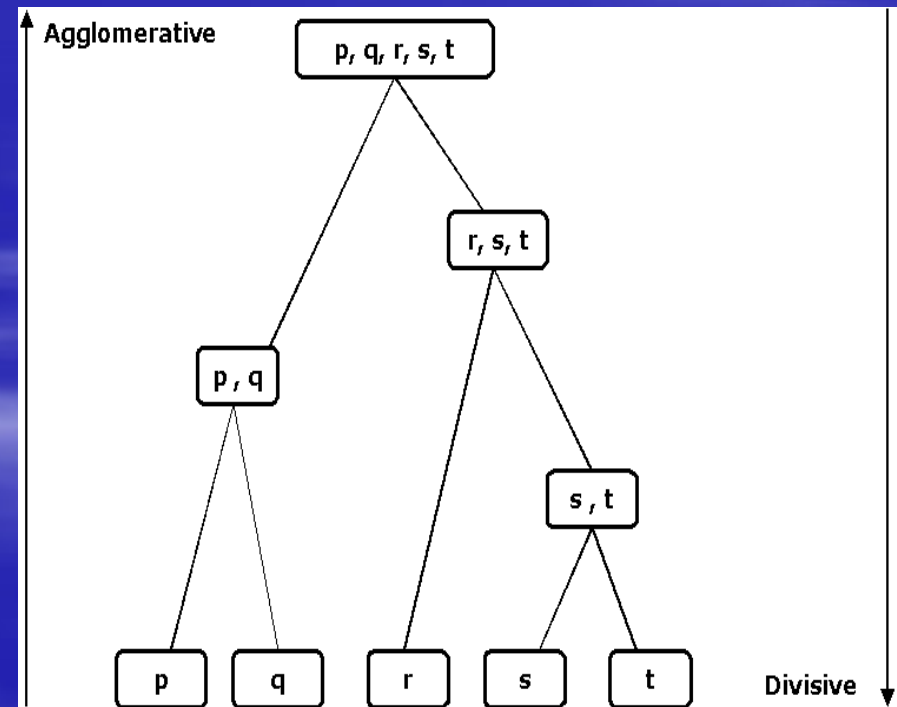
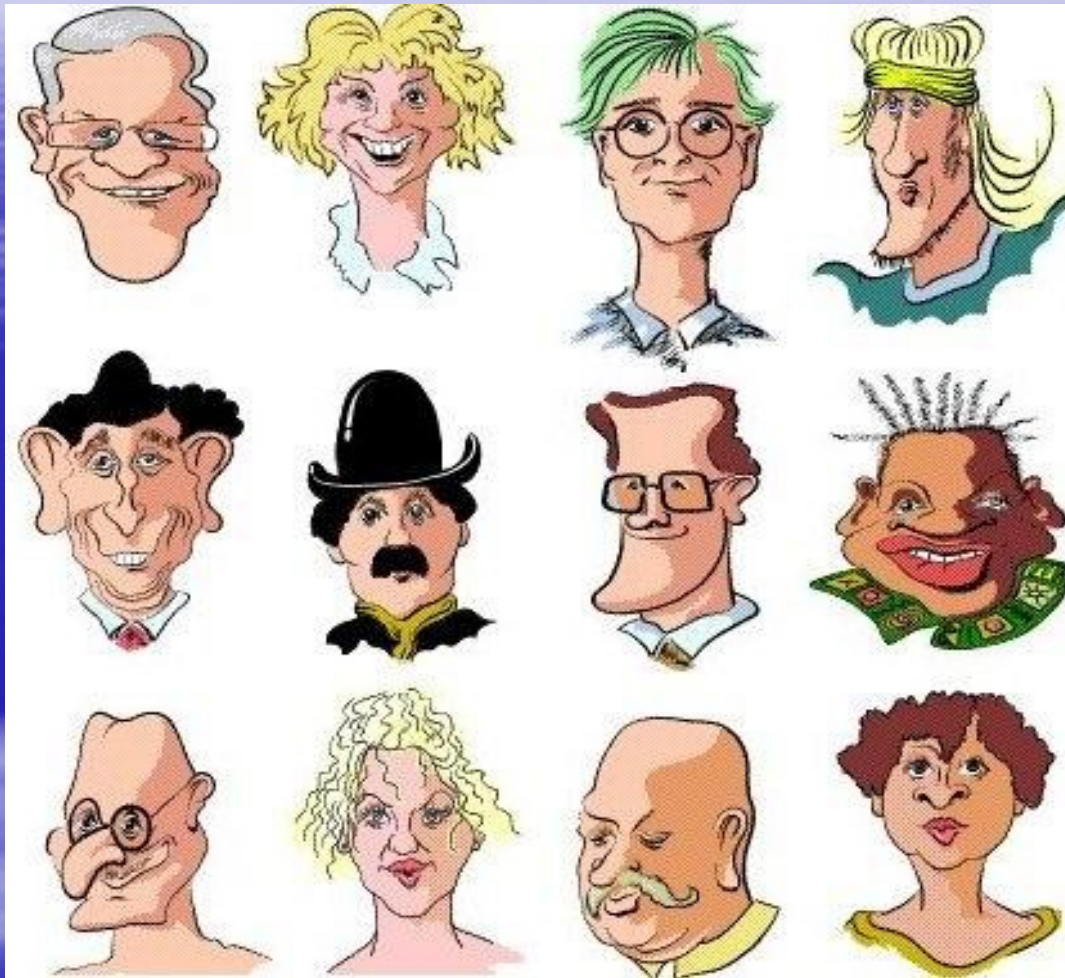


Fig. 1. Example of dendrogram

Example of Paterns:



Attributes:

Inaltime

Greutate

Culoarea ochilor

Lungimea părului

și altele

# Metode de definire a distanței (similarității) dintre clustere (grupe)

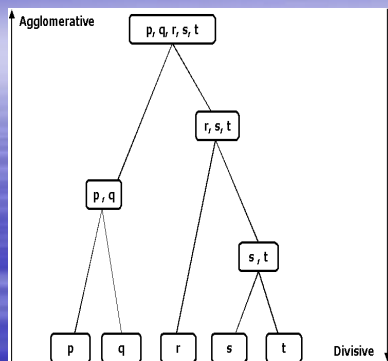


Fig. 1. Exemplu de dendrogramă

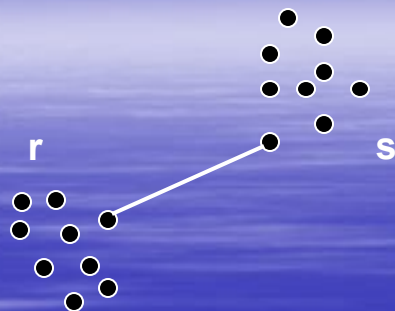


Fig. 2. Distanța minimă

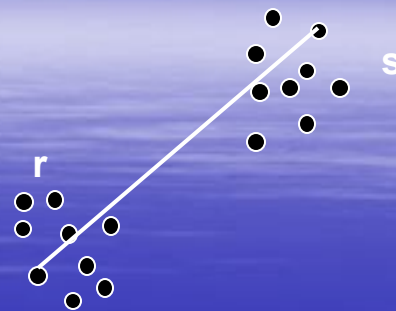


Fig. 3. Distanța maximă

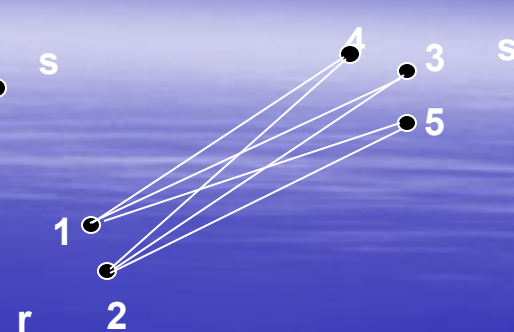


Fig. 4. Distanța medie

## 1.1. Metoda distanței medii.

$$D(r,s) = \text{Min} \{d(i,j) : i \in r \text{ și } j \in s\}$$

## 1.2. Metoda distanței maxime.

$$D(r,s) = \text{Max} \{d(i,j) : i \in r \text{ and } j \in s\}$$

## 1.3. Metoda distanței medii.

$$D(r,s) = \text{Trs} / (Nr * Ns)$$

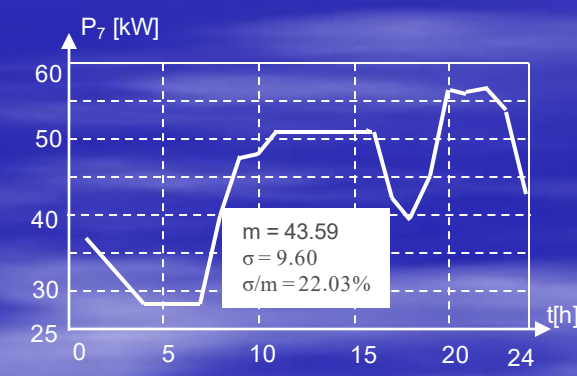
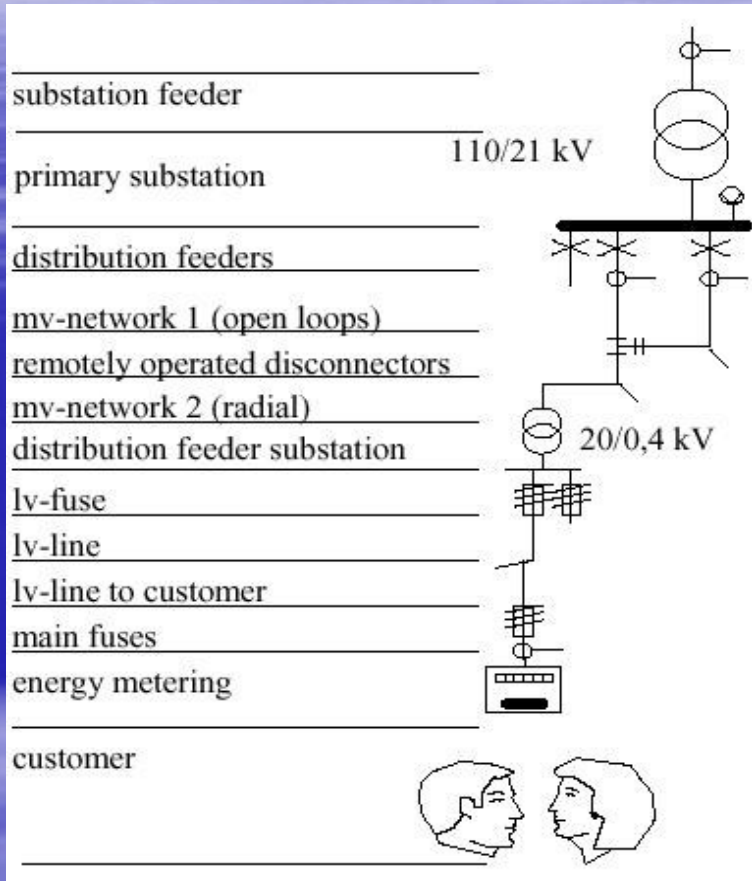
## 1.4. Metoda centrului de greutate

$$D_{KL} = \|\bar{X}_K - \bar{X}_L\|^2$$

## 1.5. Metoda k-Medii

$$J = \sum_{j=1}^k \sum_{i=1}^n \|x_i^{(j)} - c_j\|^2$$

### 3. Gruparea distribuitorilor

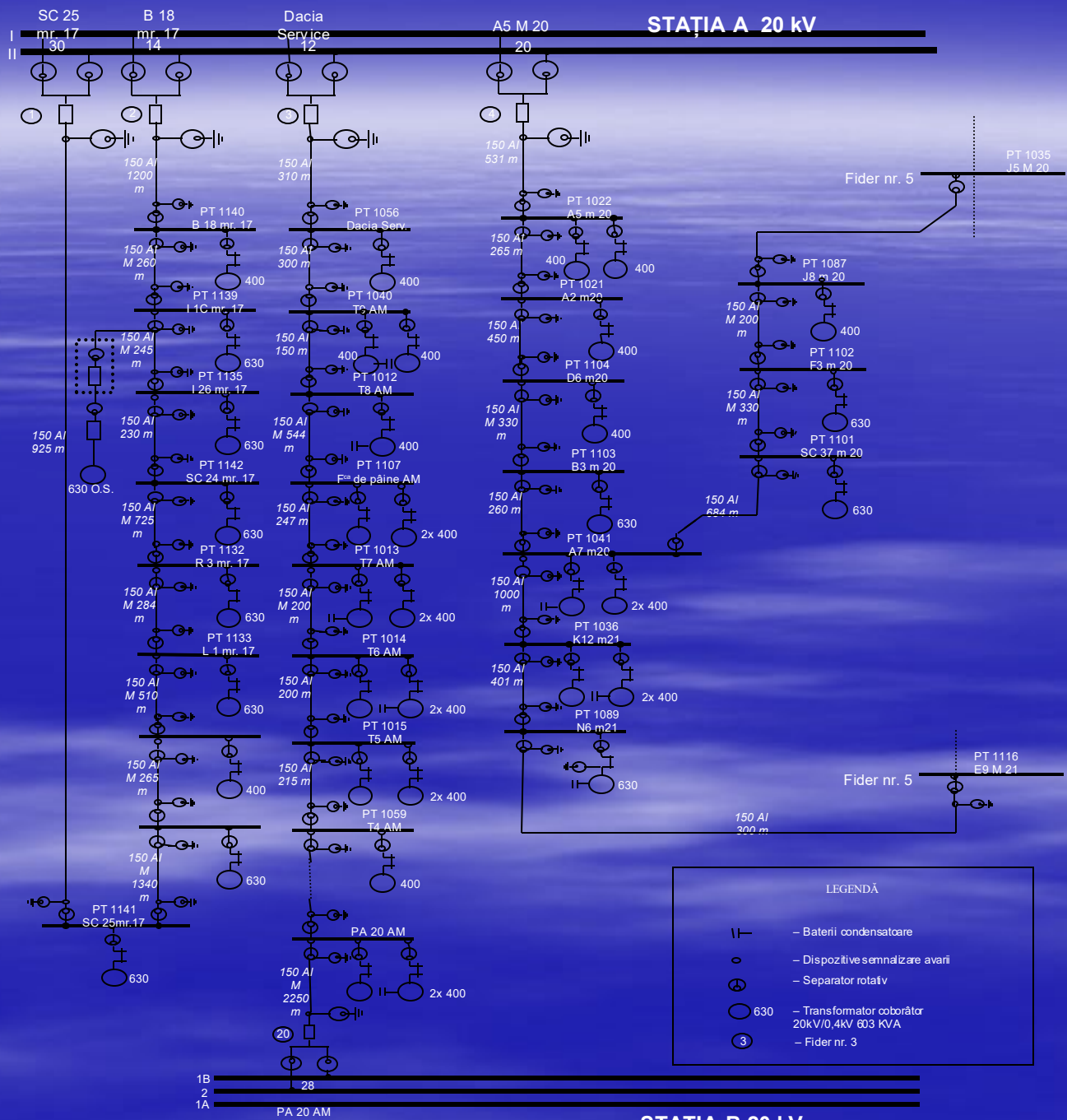


Variația puterii active în nodurile sistemului  $P(t)$

$\Delta W \sim 5\% W$

vs

10 000 distribuitori



LEGENDĂ	
	- Baterii condensatoare
	- Dispozitive semnalizare avarii
	- Separator rotativ
	- Transformator coborâtor 20kV/0,4kV 603 KVA
	- Fider nr. 3

STAȚIA B 20 kV



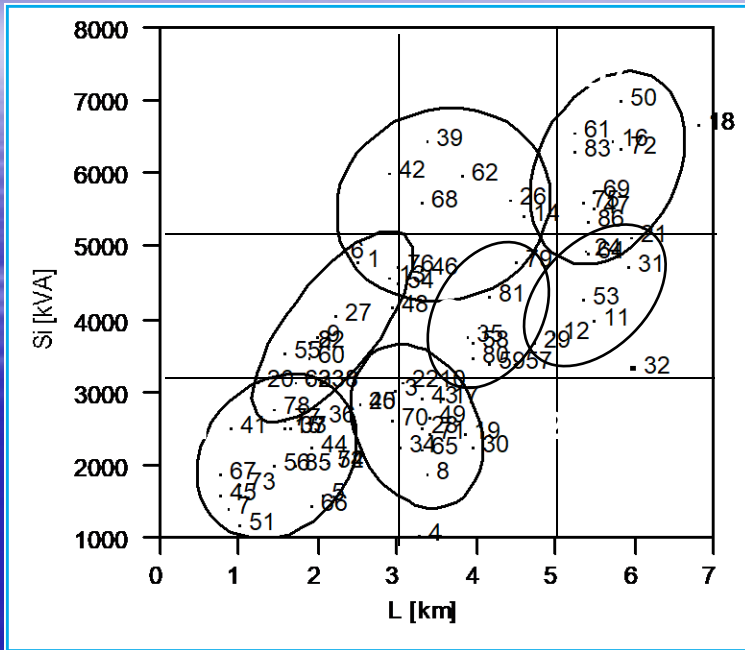
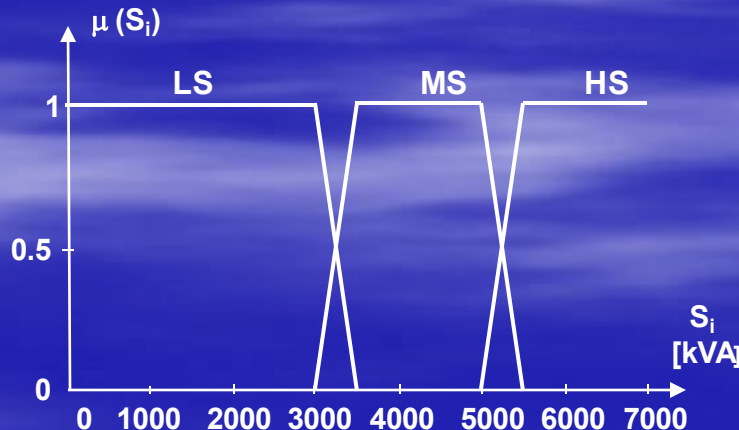
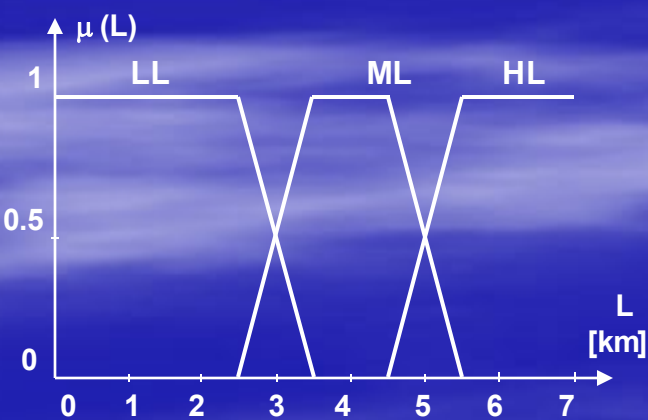


Table 4. Parametrii statistici pentru L and Si

Group	Length (L) [km]		Installed Power (Si) [kVA]	
	$m_L$	$d_L$	$m_{S_i}$	$d_{S_i}$
G1	1.49	0.47	2132.00	525.93
G2	3.26	0.39	2552.00	524.56
G3	2.31	0.55	4013.00	633.31
G4	4.23	0.32	3840.00	490.19
G5	5.50	0.35	4301.43	616.70
G6	3.75	0.67	5871.00	373.71
G7	5.67	0.45	6088.18	617.70

Gruparea distribuitorilor

Categoriile lingvistice pentru L and Si

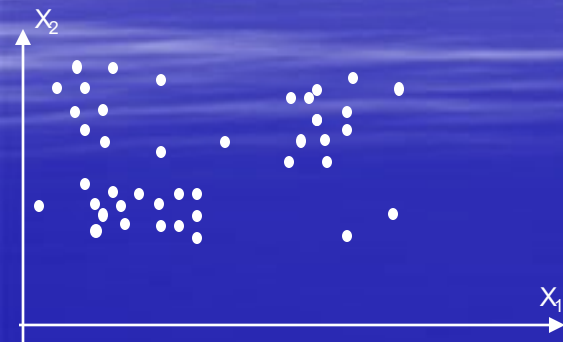


Groups	Linguistic Categories	
	L	$S_i$
G1	LL	LS
G2	ML	LS
G3	LL	MS
G4	ML	MS
G5	HL	MS
G6	ML	HS
G7	HL	HS

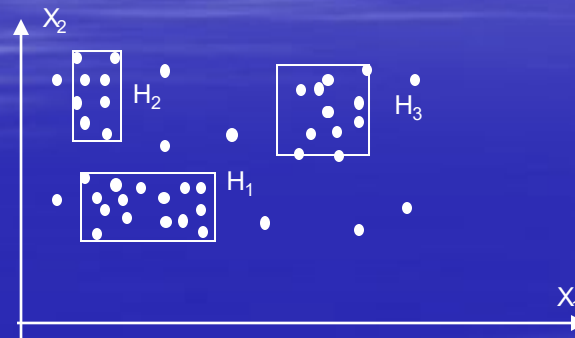
Funcțiile de apartenență pentru lungime

Funcțiile de apartenență pentru Si

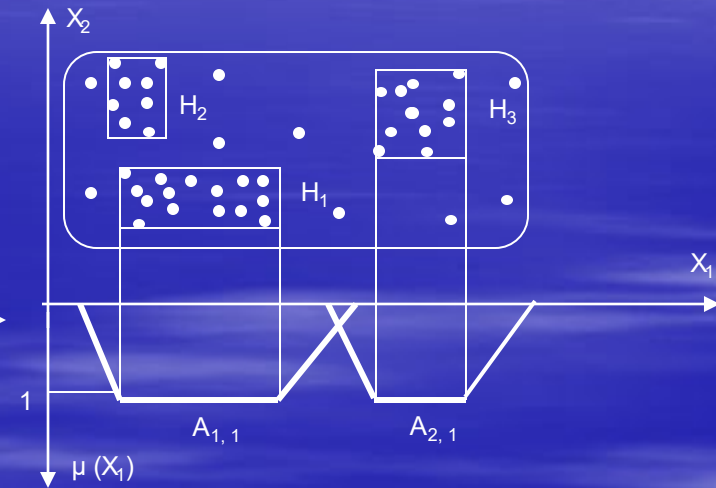
Pasii care trebuie parcursi pentru definirea functiilor de apartenenta cu ajutorul tehnicilor de apartenenta.



Elemente negrupate



Elemente grupate cu tehncile de clustering



Definirea functiilor de apartenenta

## ❑ 2. Functii de apartenenta in modelarea fuzzy

- ◆ Notiunea de modelare este esentiala in conducerea proceselor electroenergetice (sarcini, cost).
- ◆ Modelarea poate fi asigurata prin intermediul tehnicilor clasice, dar si a tehnicilor fuzzy.

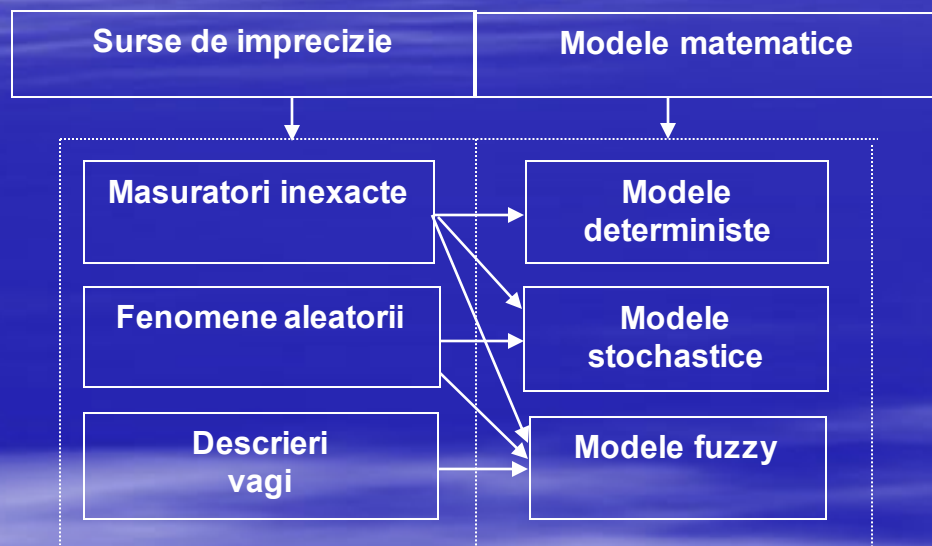
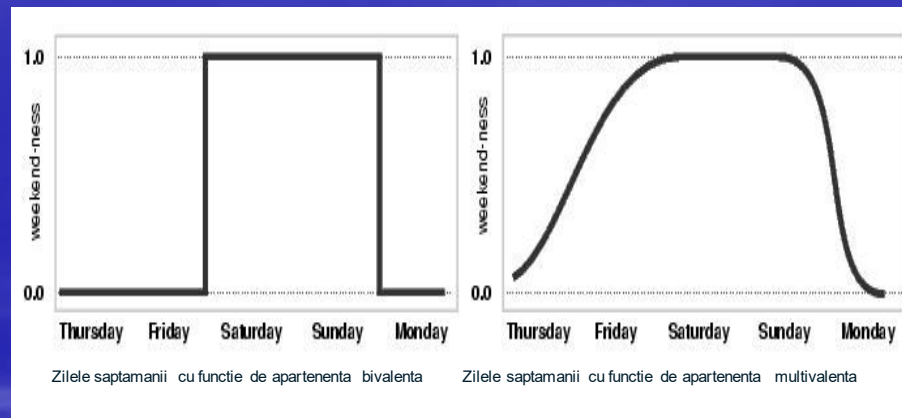


Fig. 6. Mathematical models for imprecision

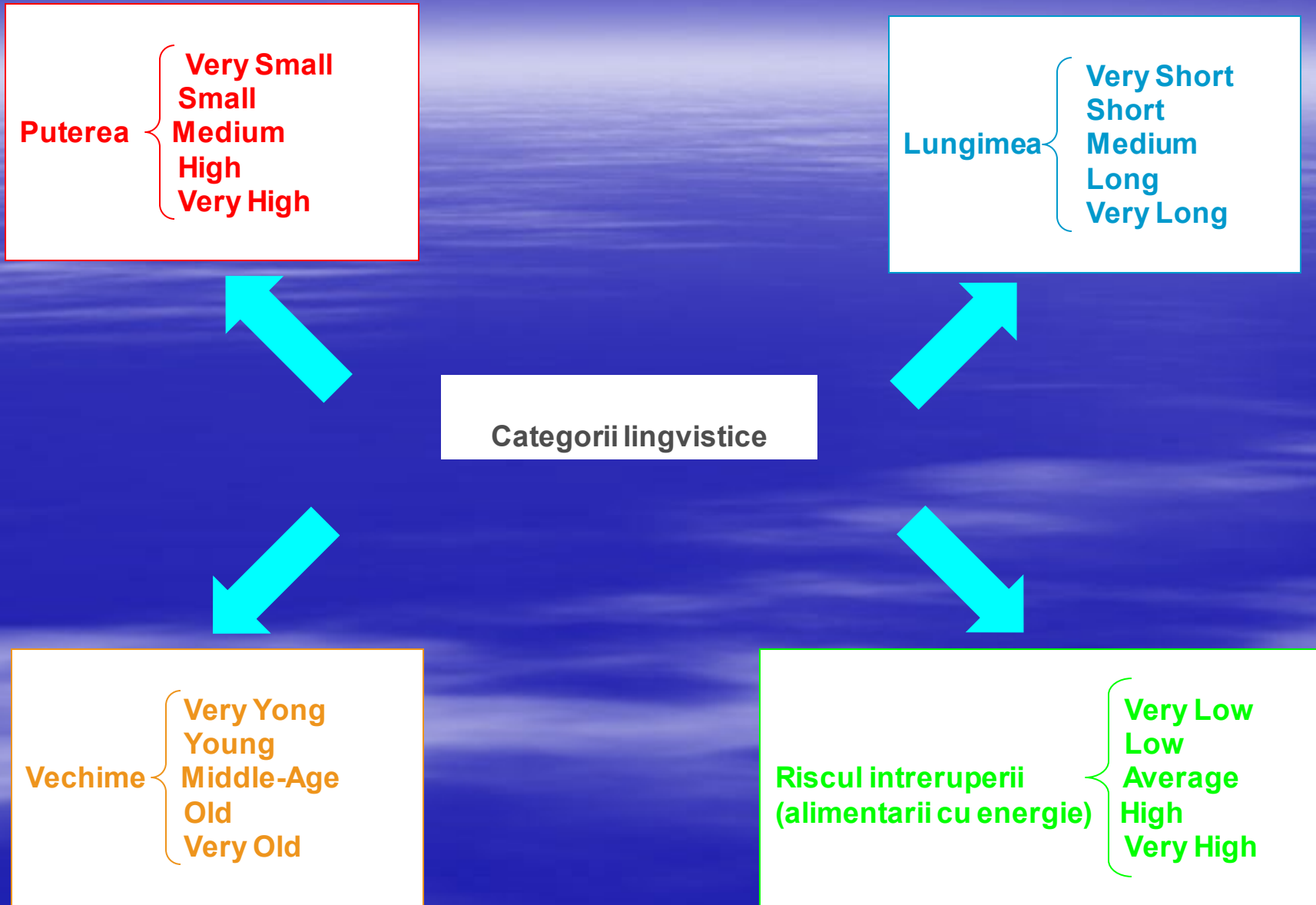


Logica Bivalent (DA – NU)

Logica multivalenta

Mic, mediu, mare sunt descrieri vagi (incerte) folosite in modelare proceselor.

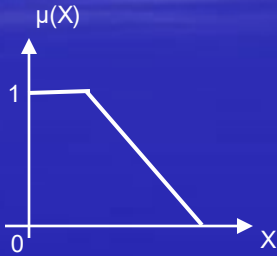
# IMPROVING OF THE FUZZY MODELS BY CLUSTERING TECHNIQUES. APPLICATIONS IN POWER SYSTEMS



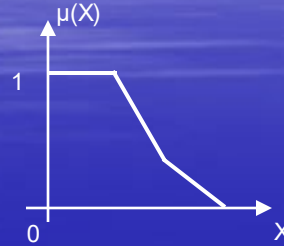
A fuzzy number can have different forms but, generally, this is represented as trapezoidal or triangular form:

$$\tilde{A} \Leftrightarrow (x_1, x_2, x_3) = [m, a, b] \quad (1)$$

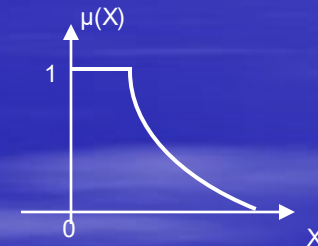
$$\tilde{A} \Leftrightarrow (x_1, x_2, x_3, x_4) = [m, n, a, b] \quad (2)$$



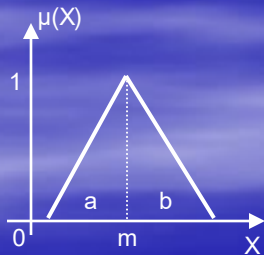
**Fig. 7. Linear membership function**



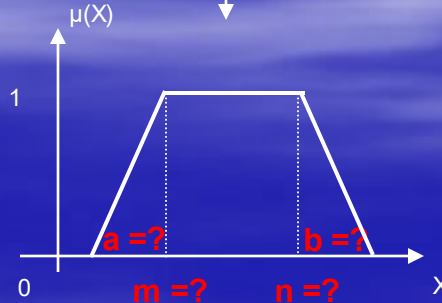
**Fig. 8. Piecewise-linear membership function**



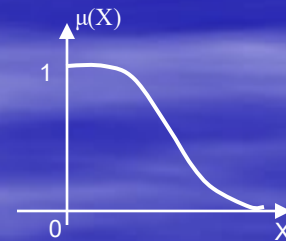
**Fig. 9. Hyperbolic membership function**



**Fig. 10. Triangular membership function**



**Fig. 11. Trapezoidal membership function**



**Fig. 12. Gaussian membership function**

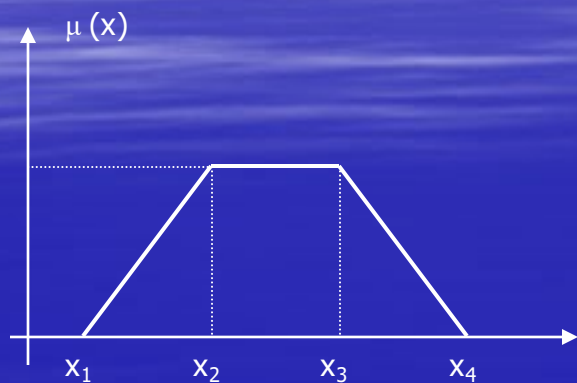


## 4. Modelarea fuzzy a sarcinilor din rețelele electrice de distribuție

- ◆ In rețelele electrice de distribuție, cu excepția măsurătorilor din posturile de transformare, există puține informații cu privire la valorile sarcinilor;

### ◆ DATE DE INTRARE

- ◆ factorul de încărcare  $kl$  and the factorul de putere  $\cos\phi$  vor fi divizate în 5 categorii lingvistice:



Modelul fuzzy pentru P și Q

1. Very Small – VS:  $kl - VS$  și  $\cos\phi - VS$ ;
2. Small – S:  $kl - S$  și  $\cos\phi - S$ ;
3. Medium – M:  $kl - M$  și  $\cos\phi - M$ ;
4. High – H:  $kl - H$  și  $\cos\phi - H$ ;
5. Very High – VH:  $kl - VH$  și  $\cos\phi - VH$ .

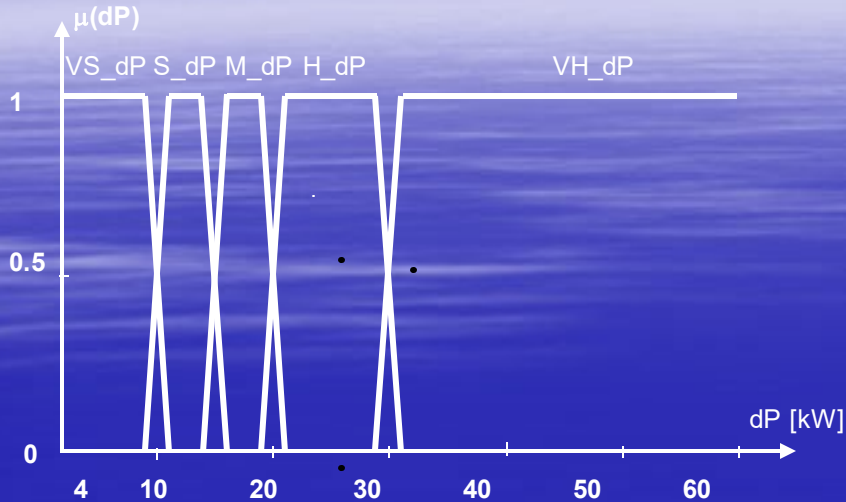
### Loading levels as function of $kl$ and $\cos\phi$

Categoria lingvistica	x		Categoria lingvistica	x			
	$kl(\%)$	$\cos\phi$		$kl(\%)$	$\cos\phi$		
VS	$x_1$	10	0.75	M	$x_3$	55	0.87
	$x_2$	10	0.77		$x_4$	65	0.89
	$x_3$	15	0.79	H	$x_1$	55	0.87
	$x_4$	25	0.81		$x_2$	65	0.89
S	$x_1$	15	0.79	VH	$x_3$	75	0.91
	$x_2$	25	0.81		$x_4$	85	0.93
	$x_3$	35	0.83	VH	$x_1$	75	0.91
	$x_4$	45	0.85		$x_2$	85	0.93
M	$x_1$	35	0.83	VH	$x_3$	95	0.95
	$x_2$	45	0.85		$x_4$	95	0.97

### Valorile medii [kW] și abaterile standard [kW] pentru pierderile de putere în funcție de nivelul de încărcare

Cluster	Nivelul de încărcare									
	VS		S		M		H		VH	
	m	d	m	d	m	d	m	d	m	d
G1	7.65	1.89	8.45	2.09	9.99	2.53	12.02	3.03	16.92	4.32
G2	9.43	2.13	10.64	2.37	12.58	2.87	15.05	3.47	21.38	4.99
G3	13.91	1.42	15.54	1.65	18.72	2.17	22.55	2.15	32.41	3.18
G4	14.01	2.57	15.60	2.89	18.52	3.37	22.54	4.19	32.58	6.59
G5	15.91	1.96	17.86	2.31	21.30	3.06	26.24	4.04	38.70	6.49
G6	20.81	2.19	23.39	2.62	28.39	3.47	34.89	4.58	50.78	7.36
G7	21.48	2.36	25.10	2.64	31.22	3.40	39.20	4.47	58.75	7.12

## 5. Estimarea pierderilor de putere cu Logica fuzzy



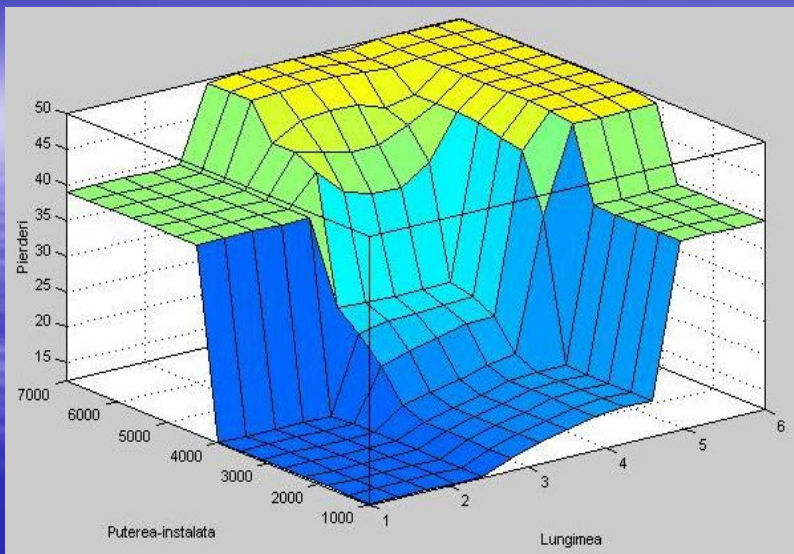
Categoriile lingvistice pentru pierderile de putere

- Very Small** - VS\_dP (4 – 10 kW),
- Small** - S\_dP (10 – 15 kW),
- Medium** - M\_dP (15 – 20 kW),
- High** - H\_dP (20 – 30 kW)
- Very High** - VH\_dP (30 – 60 kW).

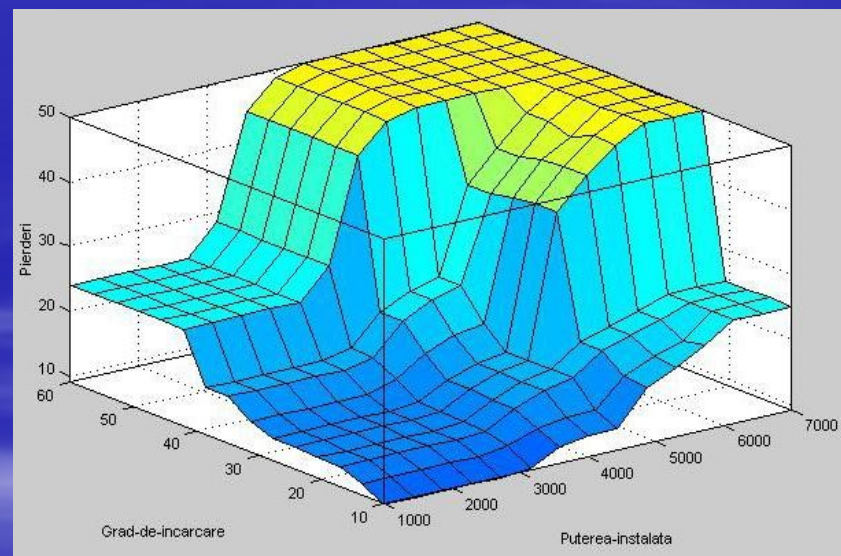
Funcțiile de apartenență pentru pierderile de putere

Tabelul fuzzy pentru estimarea pierderilor de putere în cazul rețelelor urbane

		↓ L								
		LL			ML			HL		
<b>GI</b>	<b>VL</b>	VS_dP	S_dP		VS_dP	S_dP	H_dP		M_dP	H_dP
	<b>L</b>	VS_dP	M_dP		S_dP	M_dP	H_dP		M_dP	H_dP
	<b>M</b>	VS_dP	M_dP		S_dP	M_dP	H_dP		H_dP	VH_dP
	<b>H</b>	S_dP	H_dP		M_dP	H_dP	VH_dP		H_dP	VH_dP
	<b>VH</b>	M_dP	VH_dP		H_dP	VH_dP	VH_dP		VH_dP	VH_dP
		<b>LS</b>	<b>MS</b>	<b>HS</b>	<b>LS</b>	<b>MS</b>	<b>HS</b>	<b>LS</b>	<b>MS</b>	<b>HS</b>
		S <sub>i</sub> ↑								

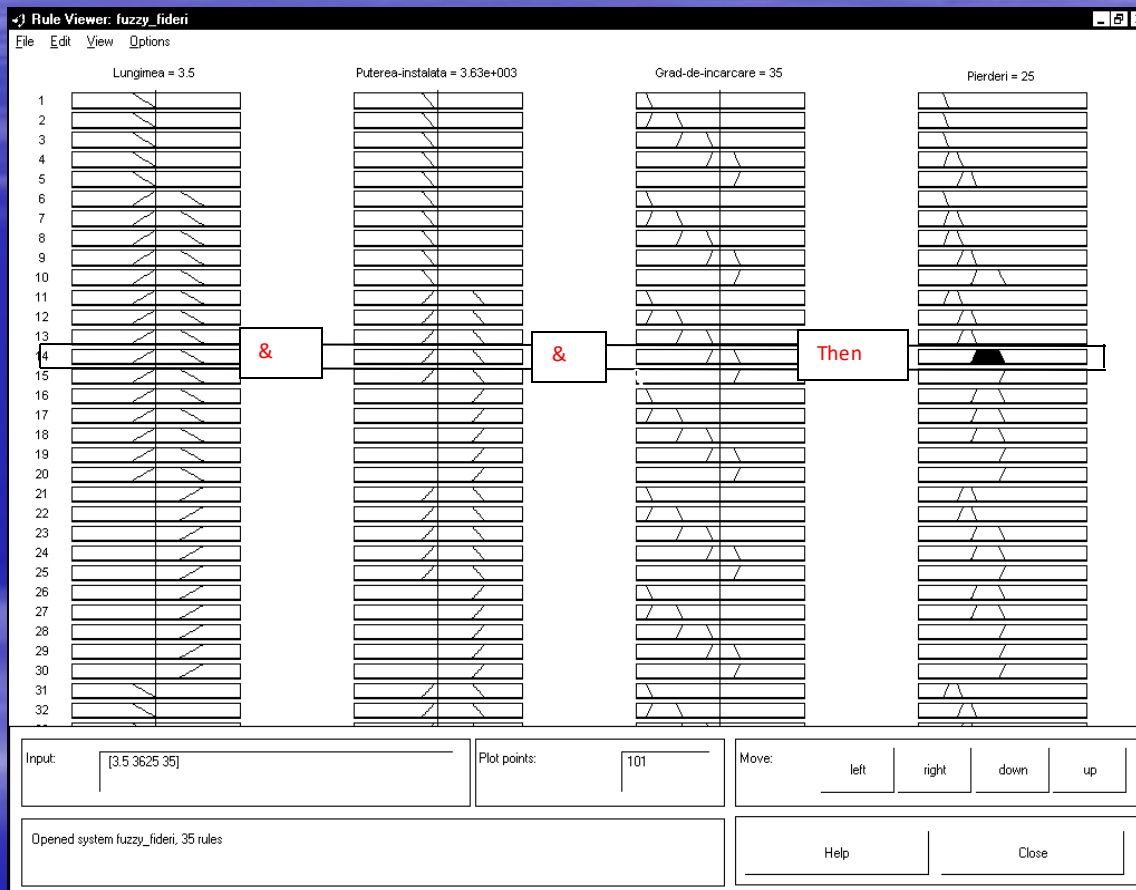


Valoarea pierderilor într-o rețea de distribuție în funcție de lungimea distribuitorilor și puterea instalată



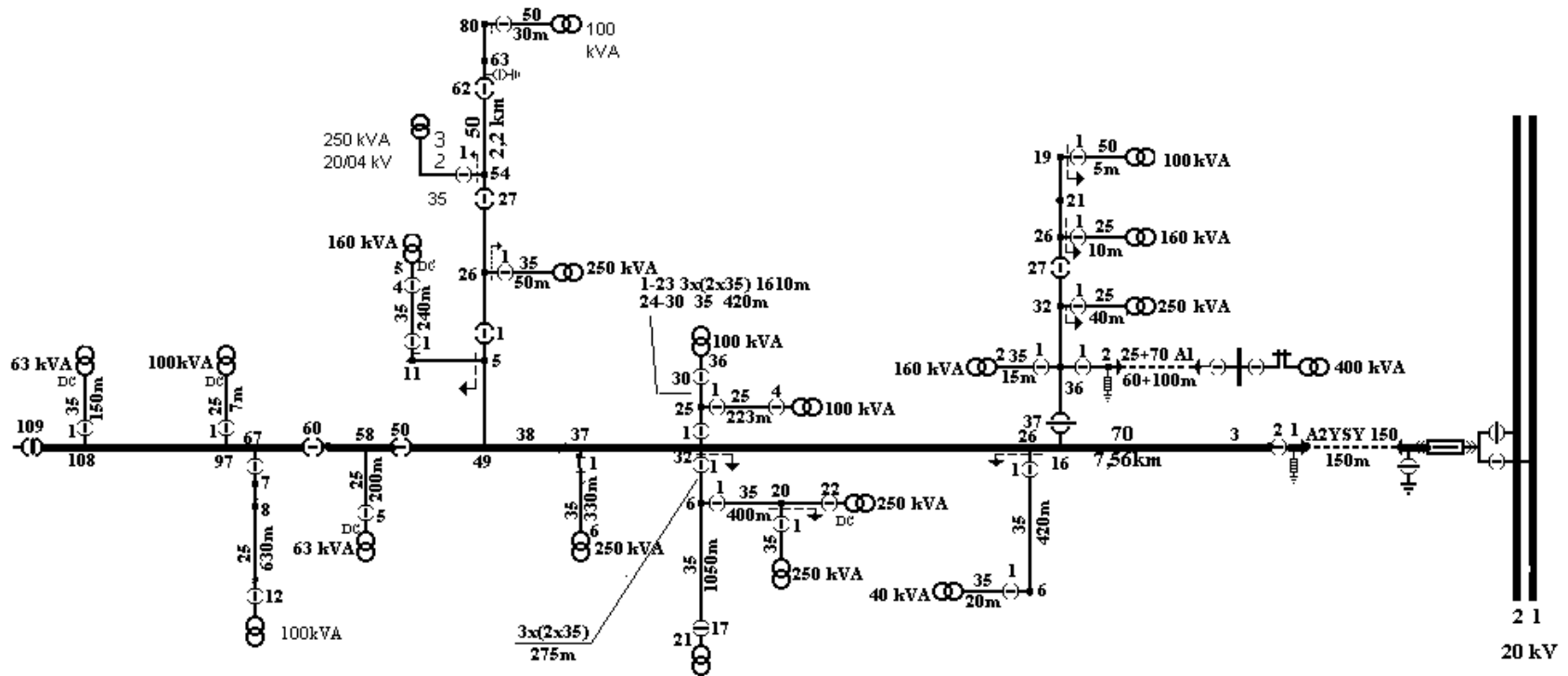
Valoarea pierderilor într-o rețea de distribuție în funcție de puterea instalată și gradul de încărcare al distribuitorilor





Estimarea fuzzy a pierderilor de putere ( $L=3,5$  km,  $S_1=3625$  [kVA],  $G1=35$ [%])

# Gruparea distribuitorilor rurali



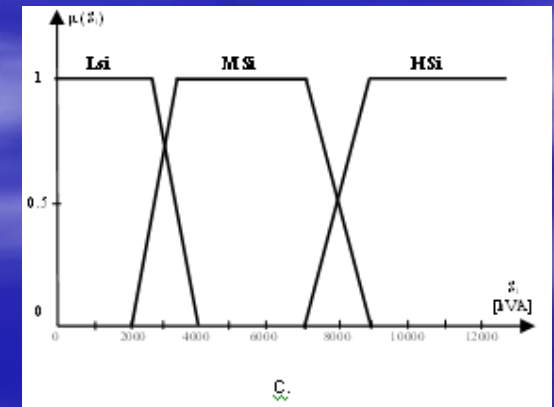
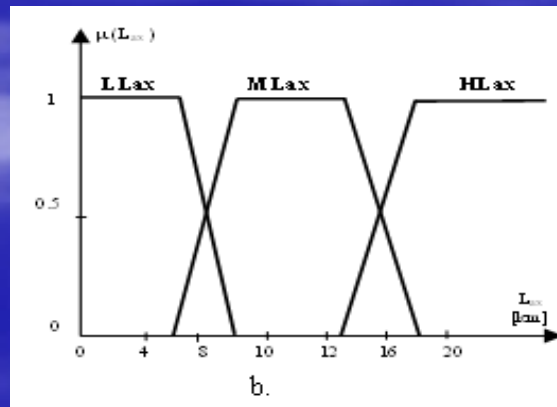
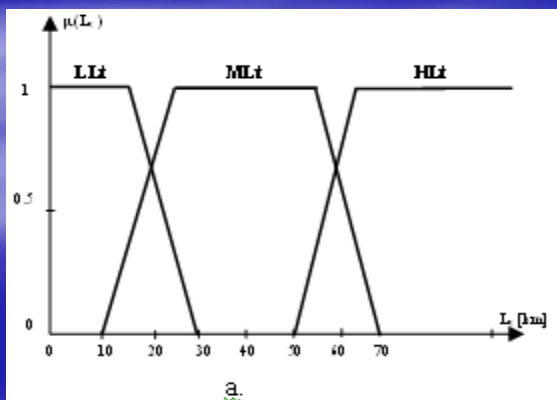
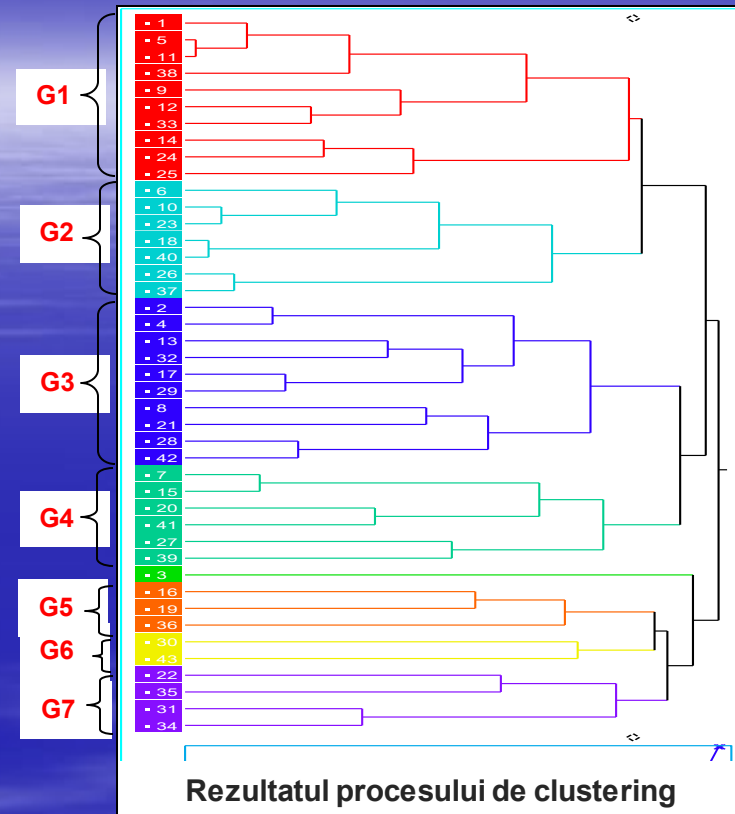
Reprezentarea simplificata a unui distribuitor rural

### Valorile medii si dispersia pentru Lt (a), Lax (b) and Si (c)

Group		Lt [km]		Lax [km]		Si [kVA]	
		$m_{Lt}$	$\sigma_{Lt}$	$m_{Lax}$	$\sigma_{Lax}$	$m_{Si}$	$\sigma_{Si}$
G1	10	6.92	4.19	4.50	2.28	2101.60	1663.95
G2	7	19.12	5.57	9.04	2.68	2405.86	639.81
G3	10	24.08	10.08	11.45	2.24	5377.60	1015.34
G4	6	17.23	6.86	7.80	3.04	10126.83	1891.82
G5	3	67.65	16.84	7.48	1.90	8572.00	1058.46
G6	2	85.48	25.62	20.70	7.78	11262.50	3483.03
G7	4	57.34	22.93	21.90	8.53	5023.50	3318.62

### Funcțiile de apartenență pentru Lt (a), Lax (b) and Si (c)

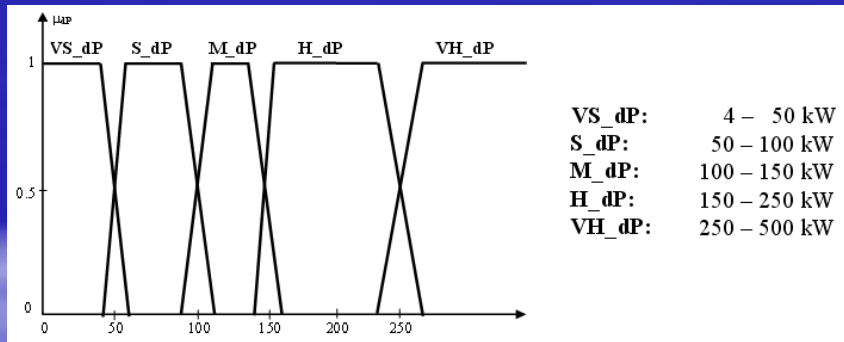
Groups	Linguistic Categories		
	$L_t$	$L_{ax}$	$S_i$
G1	LLt	LLax	LSi
G2	LLt	MLax	LSi
G3	MLt	MLax	MSi
G4	MLt	SLax	HSi
G5	HLt	SLax	HSi
G6	HLt	HLax	HSi
G7	MLt	HLax	MSi



Funcțiile de apartenență pentru Lt (a), Lax (b) and Si (c)

### Valorile medii si dispersia pentru Lt (a), Lax (b) and Si (c)

Group	Loading Level									
	VS		S		M		H		VH	
	m	$\sigma$	m	$\sigma$	m	$\sigma$	m	$\sigma$	m	$\sigma$
G1	12.77	12.70	14.23	13.59	17.02	15.67	20.67	18.87	29.66	28.02
G2	25.28	16.16	28.03	16.59	33.29	18.00	40.14	20.88	57.22	31.16
G3	46.20	18.34	53.01	18.29	65.96	18.86	83.00	21.24	125.35	32.04
G4	60.49	36.27	76.31	42.71	106.76	57.06	146.81	78.54	225.01	136.40
G5	107.26	42.87	122.87	43.86	153.08	46.32	193.07	50.02	295.72	60.53
G6	111.95	35.22	139.32	41.50	192.41	56.01	263.15	77.49	447.99	137.34
G7	73.86	36.40	86.99	43.92	111.89	59.65	144.96	81.82	233.14	142.97



Functiile de apartenenta pentru pierderile de putere

### Tabelul fuzzy pentru estimarea pierderilor de putere in cazul retelelor urbane

		Total Length									Installed Power
		LLt			MLt			HLt			
		dP_VS	dP_VS		dP_S	dP_S		dP_M	dP_M		
VS	dP_S				dP_S	dP_S					MS <sub>i</sub>
	dP_VS	dP_VS						dP_M		dP_M	HS <sub>i</sub>
S	dP_VS	dP_VS			dP_S	dP_S					LS <sub>i</sub>
	dP_S							dP_M		dP_M	HS <sub>i</sub>
M	dP_VS	dP_VS									LS <sub>i</sub>
	dP_M				dP_S	dP_M					MS <sub>i</sub>
H	dP_VS	dP_S						dP_M		dP_H	HS <sub>i</sub>
	dP_M							dP_H		dP_VH	HS <sub>i</sub>
VH	dP_VS	dP_VS									LS <sub>i</sub>
	dP_H				dP_M	dP_H					MS <sub>i</sub>
		dP_VH						dP_VH		dP_VH	HS <sub>i</sub>
		LLax	MLax	HLax	LLax	MLax	HLax	LLax	MLax	HLax	
		Axe Length									

- Evaluarea pierderilor

$$\Delta W_T = (\Delta P_L + \Delta P_{TrCo}) \cdot LF \cdot 8760 + \Delta P_{TrIr} \cdot 8760$$

unde  $\Delta P_L$  – pierderile de putere in cabluri,  
 $\Delta P_{TrCo}$  – pierderile in Cu,  
 $\Delta P_{TrIr}$  – pierderile de Fe,  
LF – factor de pierderi.

$$LF = \left( 0.124 + \frac{T_{\max}}{10000} \right)^2$$
$$T_{\max} = \frac{\sqrt{W_P^2 + W_Q^2}}{S_{\max}}$$

where:  $W_P$  – energia activa pe perioada de studiu,  
 $W_Q$  - energia activa pe perioada de studiu,  
 $S_{\max}$  – sarcina de varf,  
 $T_{\max}$  – perioada sarcinii maxime.

- Gruparea distribuitorilor de MT

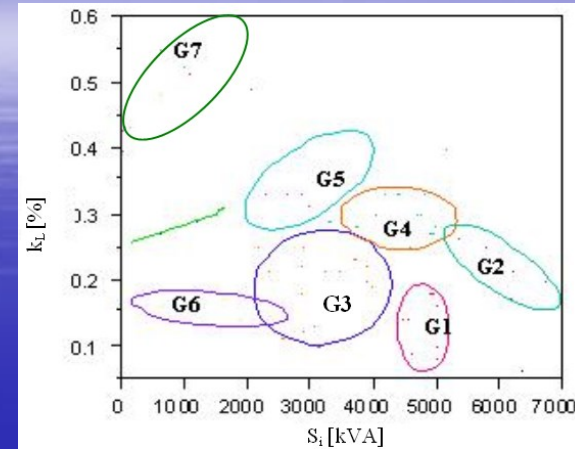
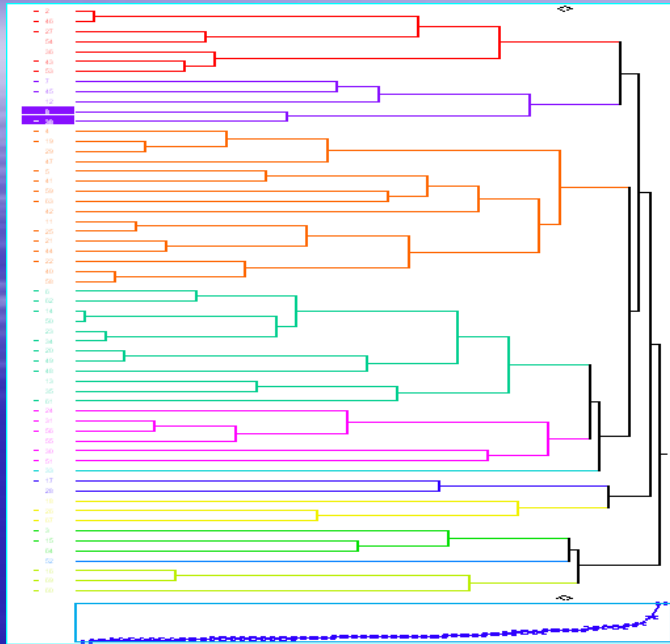


Figura 16. Rezultatele procesului de clustering

Figura 15. Dendograma procesului de clustering

Tabelul 2. Valorile statistice “m” si “σ” pentru  $k_L$ ,  $S_i$  si LF

Group	$k_L$ [%]		$S_i$ [kVA]		LF	
	$m_{kL}$	$\sigma_{kL}$	$m_{S_i}$	$\sigma_{S_i}$	$m_{LF}$	$\sigma_{LF}$
G1	0.11	0.028	4738.00	248.63	0.12	0.07
G2	0.22	0.037	6070.00	510.19	0.48	0.08
G3	0.19	0.047	3076.38	551.77	0.47	0.16
G4	0.30	0.025	4468.75	527.75	0.39	0.14
G5	0.35	0.041	2998.33	571.08	0.28	0.24
G6	0.16	0.015	1406.66	686.02	0.39	0.15
G7	0.56	0.021	915.00	403.05	0.10	0.08

Table 3. Categoriile lingvistice ale variabilelor fuzzy

$k_L$	$S_i$	LF
Very Small (VS_kL)	Very Low (VL_S)	Very Small (VS_LF)
Small (S_kL)	Low (L_S)	Small (S_LF)
Medium (M_kL)	Medium (M_S)	Medium (M_LF)
High (H_kL)	High (H_S)	High (H_LF),
Very High (VH_kL)	Very High (VH_S)	Very High (VH_LF)

Table 4. Categoriile lingvistice caracteristice grupelor obtinute

Groups	Linguistic Categories		
	$k_L$	$S_i$	LF
G1	VS	HS	S_LF
G2	S	VHS	VH_LF
G3	S	MS	VH_LF
G4	M	HS	H_LF
G5	H	MS	M_LF
G6	S	LS	H_LF
G7	VH	VLS	S_LF

## Studiu de caz

Table 5. Rezultatele obtinute pentru distribuitorii alimentati dintr-o statie electrica

Feeder	Loading level	Installed Power	Loss Factor	$dW_{Tcrisp}$ [MWh]	$dW_{Treal}$ [MWh]	Err [%]
1	S	MS	VH_LF	93.89	92.49	-1.51
2	H	MS	H_LF	185.63	186.06	0.23
3	S	HS	VH_LF	115.35	116.79	1.23
4	S	HS	VH_LF	125.64	126.50	0.68
5	M	HS	H_LF	181.34	181.48	0.08
6	H	MS	H_LF	81.58	83.10	1.83
7	S	MS	VH_LF	119.50	119.41	-0.07

# DETERMINAREA PROFILURILOR DE SARCINĂ ÎN REȚELELE ELECTRICE DE 110 KV CU AJUTORUL TEHNICILOR DE GRUPARE

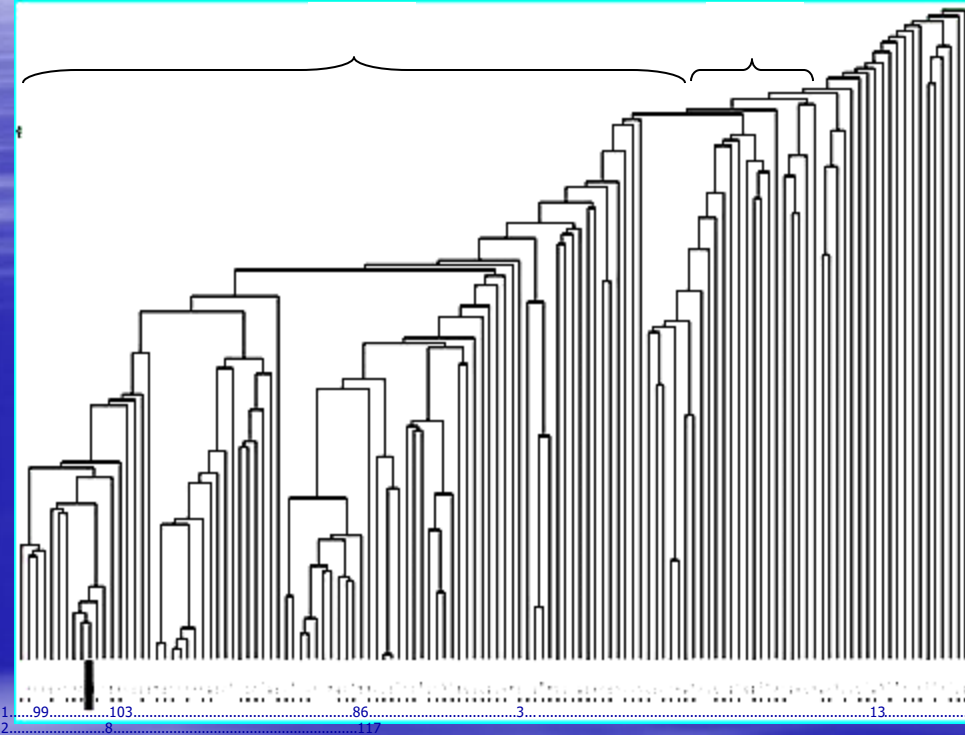


Fig. 3.12. Dendrograma pentru gruparea nodurilor după puterea activă

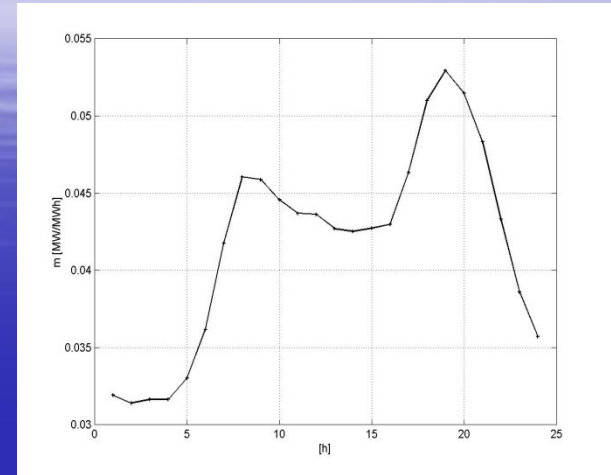


Fig. 3.14. Graficul tip de sarcină pentru grupa CP1

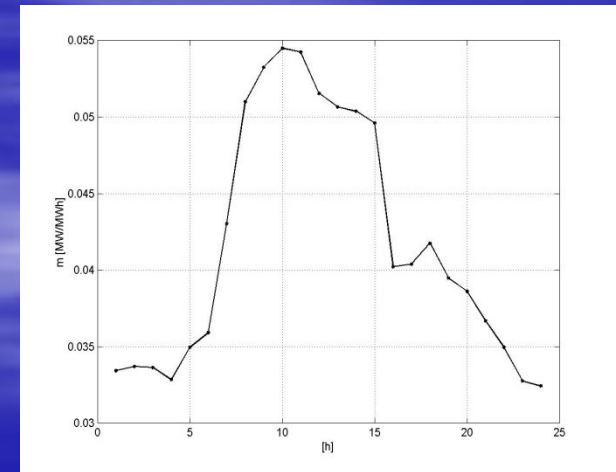


Fig. 3.15. Graficul tip de sarcină pentru grupa CP2



Tabelul 4.10. Grupele de noduri de 20 kV

Grupa	Nr.noduri	$W_{\min}$ [kWh]	$W_{\max}$ [kWh]
$C_{P1}$ – Small (S)	6	0	4500
$C_{P2}$ – Medium (M)	14	4500	9000
$C_{P3}$ – High (H)	14	9000	12000

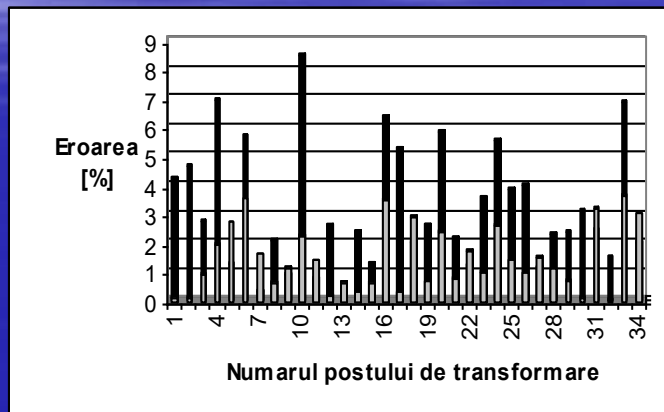


Fig. 4.29. Erorile de estimare a sarcinii din noduri, la ora vîrfului de sarcină (valori crisp)

$$[x_1, x_2, x_3, x_4] = [m - 1,28\sigma, m - \sigma, m + \sigma, m + 1,28\sigma]$$

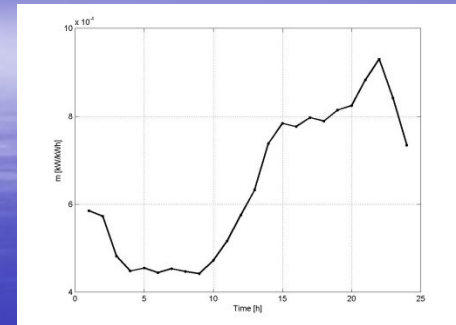


Fig. 4.26. Graficul tip de sarcină CP1

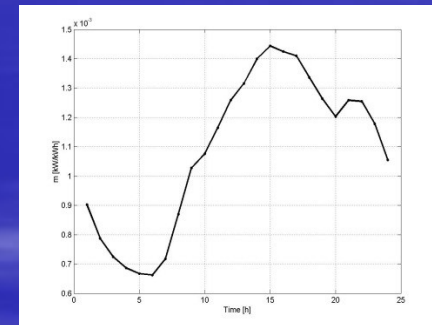


Fig. 4.27. Graficul tip de sarcină CP2

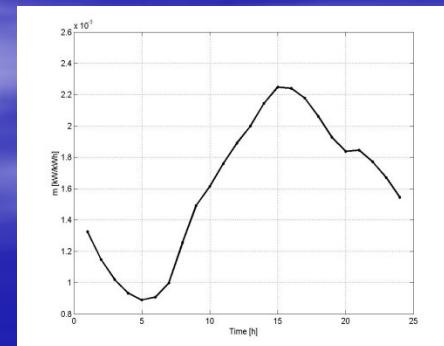


Fig. 4.27. Graficul tip de sarcină CP3

## 6. DETERMINATION OF THE OPTIMAL CONTROL INTERVALS

Minimize 
$$F([\theta],[U],[tc]) = \Delta P([\theta],[U],[tc]) + \sum_{i \in q} QGC_i(Q_i) + \frac{1}{T} \sum_{i,j \in r} TSC_{ij \in r}(\Delta tc_{ij}) = \min!$$

subject to:

$$\begin{aligned} P_i([\theta],[U],[tc]) - P_{G_i} + P_{L_i} &= 0, & i \in n \setminus e, \\ Q_i([\theta],[U],[tc]) - Q_{G_i} + Q_{L_i} &= 0, & i \in C, \\ U_{i \min} \leq U_i \leq U_{i \max}, & & i \in n \\ Q_{i \min} \leq Q_i([\theta],[U],[tc]) \leq Q_{i \max}, & & i \in q \\ tc_{ij \min} \leq tc_{ij} \leq tc_{ij \max}, & & ij \in r \\ 0 \leq I_{ij}^2([\theta],[U],[tc]) \leq I_{ij \max}^2, & & ij \in b \end{aligned}$$

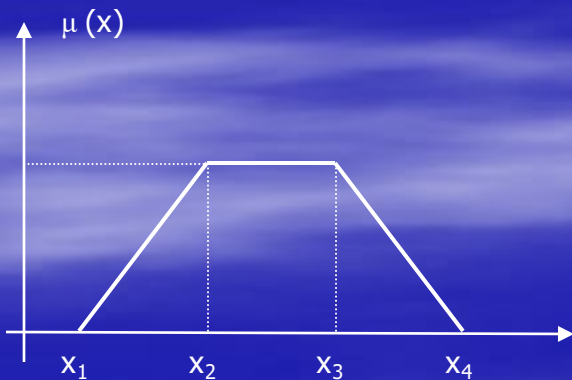
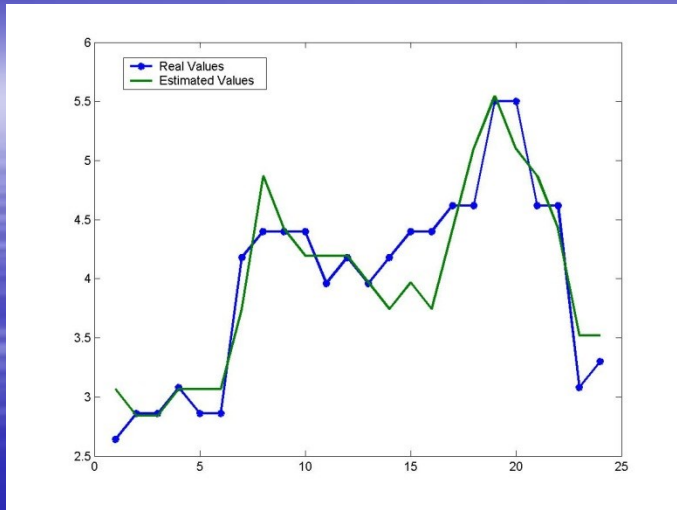


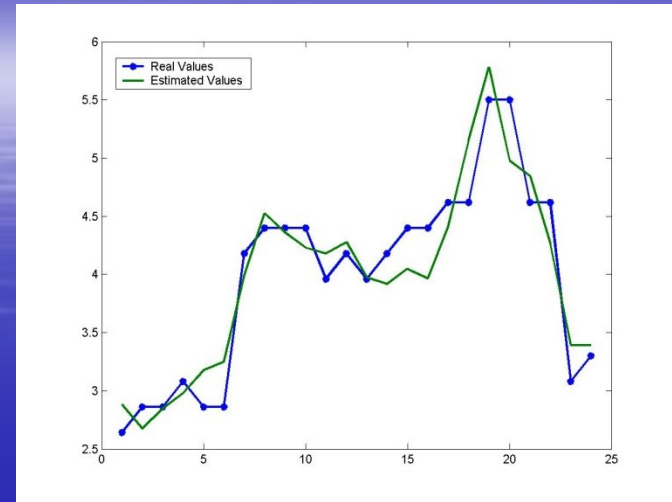
Fig. 19. Fuzzy trapezoidal model for active and reactive power

Table 2. Breaking points for hourly coefficients for CP1, CP2 and CQ groups

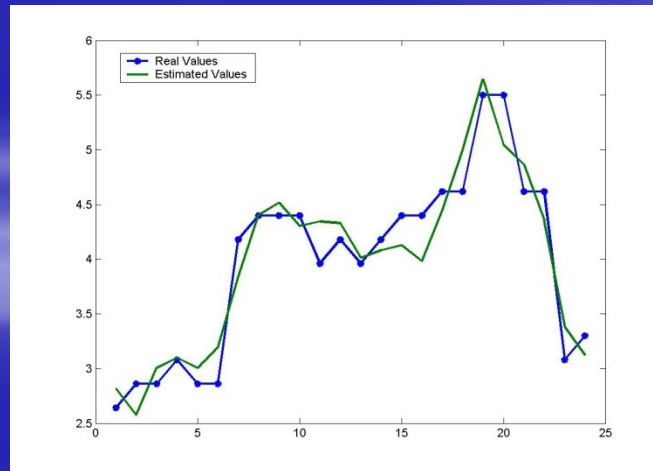
Breaking points	C <sub>P1</sub>	C <sub>P2</sub>	C <sub>Q</sub>
X <sub>1</sub>	$m_{P_1}^h - 1.1 d_{P_1}^h$	$m_{P_2}^h - 1.12 d_{P_2}^h$	$m_{Q_1}^h - 1.28 d_{Q_1}^h$
X <sub>2</sub>	$m_{P_1}^h - d_{P_1}^h$	$m_{P_2}^h - d_{P_2}^h$	$m_{Q_1}^h - d_{Q_1}^h$
X <sub>3</sub>	$m_{P_1}^h + d_{P_1}^h$	$m_{P_2}^h + d_{P_2}^h$	$m_{Q_1}^h + d_{Q_1}^h$
X <sub>4</sub>	$m_{P_1}^h + 1.1 d_{P_1}^h$	$m_{P_2}^h + 1.12 d_{P_2}^h$	$m_{Q_1}^h + 1.28 d_{Q_1}^h$



**Fig. 20. Active Power Estimation of Node 5 (a single regressor)**



**Fig. 21. Active Power Estimation of Node 5 (two regressors)**



**Fig. 22. Active Power Estimation of Node 5 (all regressors)**

Table 3. Optimal transformer taps changing positions with the fuzzy modeling

Hour	Fuzzy control variables (transformers taps)														
	T <sub>1-2</sub>					T <sub>3-6</sub>					T <sub>8-9</sub>				
	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	real	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	real	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	real
1	8	8	11	11	10	7	7	10	10	9	2	2	4	4	4
2	8	8	10	11	10	7	7	10	10	9	2	2	4	4	4
3	8	8	10	11	10	7	7	9	10	9	2	2	4	4	4
4	8	8	10	11	10	7	7	9	10	9	2	2	4	4	4
5	8	9	11	11	10	7	8	10	10	9	2	2	4	4	4
6	8	9	11	11	10	7	8	10	10	9	2	3	4	4	4
7	10	10	14	15	11	9	9	13	14	10	3	3	5	5	4
8	10	10	15	15	12	9	9	14	14	9	3	3	6	7	5
9	10	10	15	15	13	9	10	14	15	13	3	4	6	7	5
10	10	10	15	15	13	9	10	15	15	12	3	4	7	7	5
11	10	11	15	15	14	10	10	14	15	14	4	4	7	7	5
12	10	11	15	15	14	10	10	15	15	14	4	4	7	7	5
13	10	11	15	15	12	10	10	14	14	12	4	4	7	7	5
14	10	11	15	15	13	10	10	14	15	13	4	4	7	7	5
15	10	11	15	15	11	10	10	14	14	10	4	4	6	7	4
16	10	10	15	15	11	9	10	14	14	10	3	4	5	6	4
17	10	10	15	15	12	9	10	14	14	11	3	4	6	7	5
18	10	11	15	15	11	10	10	14	14	10	4	4	7	7	5
19	10	11	15	15	11	10	10	14	14	10	4	4	7	7	5
20	10	11	15	15	11	10	10	14	14	10	3	4	6	7	5
21	10	10	15	15	11	9	9	14	14	10	3	3	5	6	4
22	10	10	11	14	11	9	9	10	13	10	3	3	5	5	4
23	9	9	11	11	10	8	9	10	10	9	3	3	4	5	4
24	9	9	11	11	10	9	8	10	10	9	3	3	4	4	4

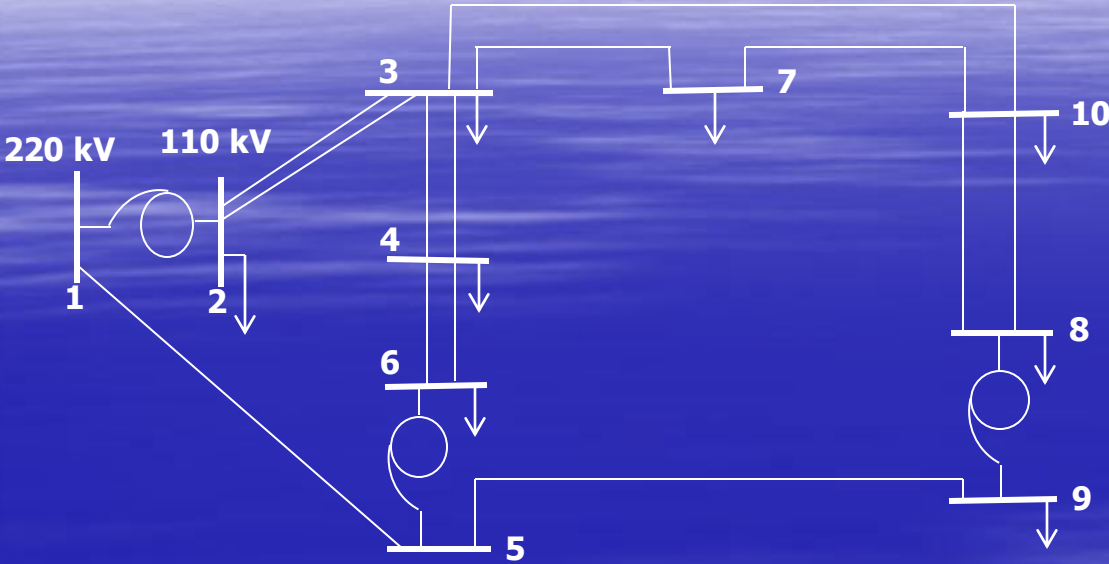


Fig. 23. Test network of 220/110 kV